유한요소해석기법을 활용한 일축대칭 변단면 I 형보의 좌굴강도 특성 고찰

A STUDY ON LATERAL TORSIONAL BUCKLING STRENGTH OF NONPRISMATIC MONOSYMMETRIC I-BEAM USING FINITE ELEMENT ANALYSIS

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Abstract.

Stepped I-beams having increased moment of inertia at one end (singly stepped beam) or both ends (doubly stepped beams) can often be seen in construction of bridges due to material economy and easy fabrication of the section. This paper presents the results of the parametric study of lateral torsional buckling of monosymmetric stepped I-beams with constant depth subjected to equal and opposite end moments applied at the end of the beam. Design recommendations were made based on the finite element results of the models having different combinations of monosymmetric ratio, stepped length ratio, flange thickness ratio and flange width ratio,. The proposed approximation is acceptable based on the parameters given having mostly conservative results. The proposed equation can be further used to extend the study to different loading conditions.

Keyword . Lateral torsional buckling, Stepped beam, Monosymmetric beam.

1. Introduction

Stepped I-beams are beams having increased moment of inertia at one end (singly stepped beam) or both ends (doubly stepped beams). This type of nonprismatic beam can often be seen in construction of bridges due to material economy and easy fabrication of the section. Stepped beams are most efficient when laterally supported because the strength of the material is used to its full extent. When it does not have adequate lateral support, its failure is often caused by lateral torsional buckling. Even so, the use of stepped beams can still be economical as long as the flange material is distributed to improve its resistance.

Many researches on the lateral torsional buckling of monosymmetric beams and prismatic beams have been published through the years. Kitipornchai and Trahair (1980) discussed the effect of monosymmetric properties such as degree of monosymmetry (ρ), monosymmetry parameter (β_x), and beam parameter (K) on the critical buckling moments of monosymmetric beams. Trahair (1993) presented graphical solutions for simply supported monosymmetric I-beams with concentrated load, uniform and varying end moments, and investigated the effect of load height on elastic critical moment. Helwig et al (1997) suggested general equations for monosymmetric I-beams subjected to single curvature and reverse curvature bending. SSRC Guide (1998) presented a solution for lateral torsional buckling of monosymmetric i-beams subjected to uniform moment along the unbraced length as:

$$M_{ocr} = \frac{\pi}{KL}\left[\sqrt{EI_yGJ}\left(B_1 + \sqrt{1 + B_2 + B_1^2}\right)\right] \tag{1}$$

in which $B_1 = \frac{\pi\beta_x}{2(KL)}\sqrt{\frac{EI_y}{GJ}}$; $B_2 = \frac{\pi^2 EC_w}{(KL)^2 GJ}$ and $\beta_x = 0.9d'\left(\frac{2I_{yc}}{I_y} - 1\right)\left[1 - \left(\frac{I_y}{I_x}\right)^2\right]$

where L = unbraced length; E = modulus of elasticity; I_y = second moment of area of section about y-axis; G = shear modulus; J = St. Venant torsional constant; K = boundary coefficient, and C_w = warping constant.

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Trahair (1993) included his study on stepped beams on the special topics. In the book, he presented a graph and an approximate solution for solving the lateral torsional buckling of beams having steps at the center. It also included the effects of step locations in the minor axis flexural rigidity, torsional rigidity and warping rigidity of beams. It was found that steps located on the flange width and flange thickness have the greatest effect on the lateral torsional buckling resistance of step beams. In 2002, Park started a series of researches on the effect of LTB on stepped beams considering two general types which are doubly stepped beams and singly stepped beams. Park and Kang (2003) suggested a design equation for the lateral torsional buckling strength of doubly stepped and singly stepped beams subjected to pure bending. Park (2002) developed equations for LTB of stepped beams with continuous top flange bracing while Park and Stallings (2003) proposed an equation applicable to various loading and boundary conditions.

2. Finite element modelling

A parametric study on the lateral torsional buckling behaviour of monosymmetric stepped beams was conducted using a finite element program ABAQUS (2007). The element used for the beams is S4R shell element which was chosen because of its flexibility and the results produced are quite accurate without consuming too much time and space.

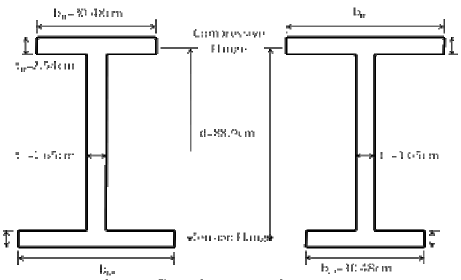
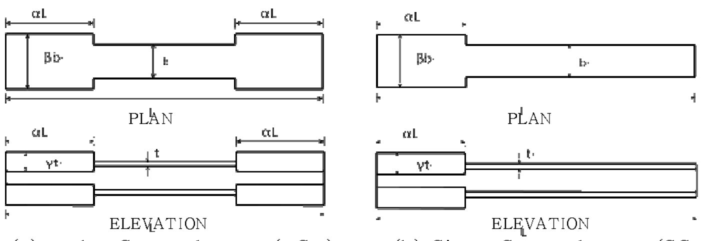


Fig 1. Section Profiles



(a)Doubly Stepped Beam (DSB)   (b) Singly Stepped Beam (SSB)

Fig 2. Definition of α , β and γ

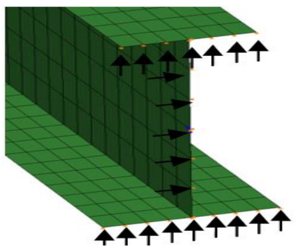


Fig 3. Boundary Condition

Table 1. Parameters for the finite element analyses

ρ	0.1, 0.3, 0.5, 0.7, 0.9
α	0.167, 0.25, 0.333 (DSB)
	0.167, 0.25, 0.333 and 0.5 (SSB)
β	1.0, 1.2 and 1.4
γ	1.0, 1.4 and 1.8

Only the monosymmetric ratios (ρ) between 0.1 and 0.9 were included in this study. Monosymmetric ratios outside of the said range have lateral torsional buckling behaviours that are hard to predict. Fig 1 shows the basic sections used. A flange size was fixed at 30.48 x 1 cm while the dimension of the other flange was varied to achieve the needed ρ values. The height and thickness of the web is fixed at 88.9 x 1.65 cm. A doubly symmetric beam has a ρ value of 0.5. A beam with monosymmetric ratio of ρ is just an inverse of a beam having a monosymmetric ratio of $1-\rho$.

Two types of stepped beams are included in this study: doubly stepped beam (DSB) and singly stepped beam (SSB). Table 1 shows the parameters used for singly and doubly stepped beams. Fig 2 defines the 3 ratios shown in Table 1 which are α (stepped length ratio), β (flange width ratio) and γ (flange thickness ratio). The ends of the beams are simply supported which are restricted to twist and deflect vertically but are allowed to warp as shown in Fig 3. The effect of L_b/h ratios was also studied. These ratios are 15, 20 and 25. Both ends of the beams are subjected to uniform but opposite moments.

3. Design recommendation

Based on the result of the parametric study, a regression program MINITAB was used to produce new design equations. For every ρ, 135 and 180 models were analyzed for doubly stepped beams and singly stepped beams, respectively.

The proposed solution for the lateral torsional buckling of monosymmetric stepped beam subjected to uniform moment along the unbraced span is

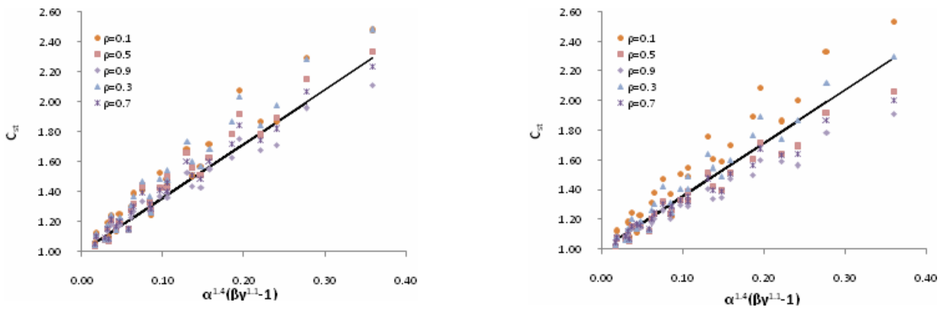
$$M_{ost} = C_{st} M_{ocr} \tag{3a}$$

with

$$C_{st} = 1 + 3.6\alpha^{1.4}(\beta\gamma^{1.1} - 1) \quad \text{for Doubly Stepped Beam} \tag{3b}$$

$$C_{st} = 1 + \alpha^{1.2}(\beta\gamma^{1.2} - 1) \quad \text{for Singly Stepped Beam} \tag{3c}$$

in which M_{ocr}=LTB strength of monosymmetric beam using Eq. 1; and α, β and γ are as defined in Fig 2.

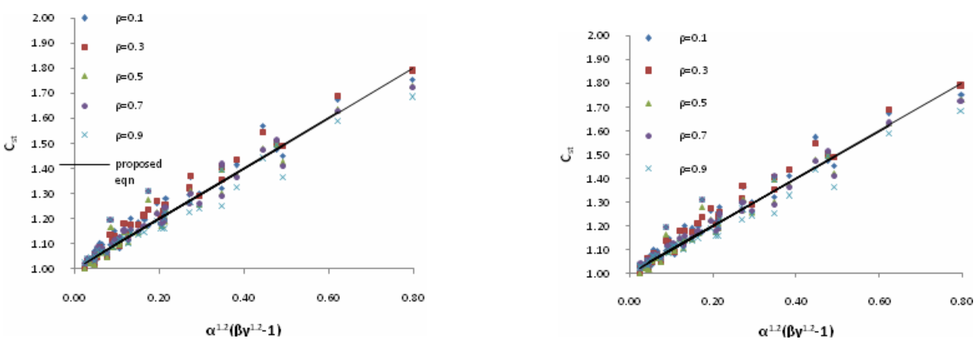


(a) $L_b/h=15$ (b) $L_b/h=25$

Fig 4. Results for Doubly Stepped Beam

Fig. 4a and 4b are comparisons between the results of finite element analyses (FEA) and the proposed equation, for $L_b/h=15$ and $L_b/h=25$, respectively. C_{st} values were plotted against $\alpha^{1.4}(\beta\gamma^{1.1}-1)$. The points represent the finite element results and the solid line represents the proposed equation. It was observed that C_{st} decreases as ρ decreases. Except for a few isolated cases, the proposed equation gives conservative results and is concurrent with the finite element results. The maximum difference of the conservative estimate is 19% with $\rho=0.1$, $\alpha=0.333$, $\beta=1$ and $\gamma=1.8$. The maximum difference of the unconservative estimate is 20% with $\rho=0.9$, $\alpha=0.333$, $\beta=1.4$, and $\gamma=1.8$.

In Figs 5a and 5b, the results of the finite element analyses (FEA) and the proposed equation are graphed, for $L_b/h=15$ and $L_b/h=25$. This graph shows that C_{st} varies linearly with $\alpha^{1.2}(\beta\gamma^{1.2}-1)$. The proposed equation produces an approximation that is acceptable within the given parameters having an error of −6% to 11%. The solution is the most conservative with $\rho=0.7$, $L_b/h=20$, $\alpha=0.5$, $\beta=1.2$, and $\gamma=1$ and the most unconservative with $\rho=0.9$, $L_b/h=15$, $\alpha=0.333$, $\beta=1.4$, and $\gamma=1.8$.



(a) $L_b/h=15$ (b) $L_b/h=25$

Fig 5. Results for Singly Stepped Beam

4. Conclusion

The results from the numerical studies using finite element analyses have been presented for doubly stepped beams with varying degree of monosymmetry ($0.1 \leq \rho \leq 0.9$). A uniform moment was applied along the length of the beam. Using the FEA results, a design recommendation for the lateral torsional buckling of monosymmetric doubly and singly stepped beams were developed. It was observed that the C_{st} decreases as the monosymmetric ratio decreases and the lateral torsional buckling increases as the span to height ratio increases. With the exemption of few isolated cases, the suggested equations generally produce conservative results and were in agreement with the FEA results. The proposed equation can be further used to extend the study to different loading conditions.

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